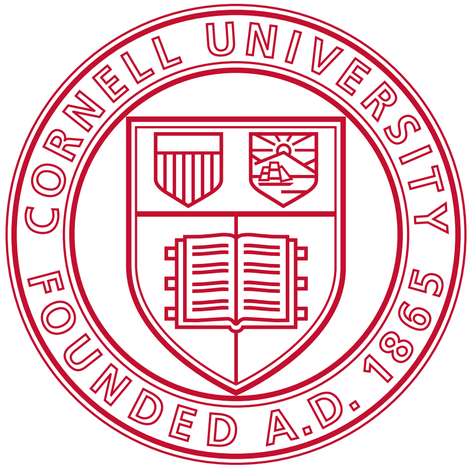
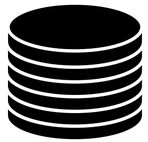
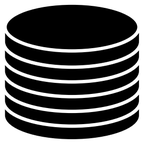
Leveraging Quantum Computing for Optimal Data Allocation in Distributed Systems

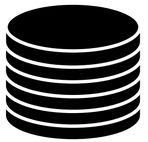
Immanuel Trummer

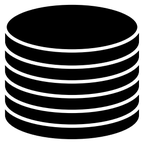


Data

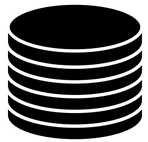




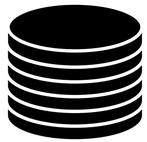




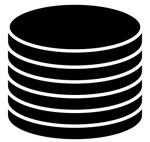
Data



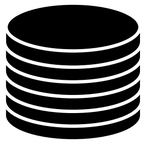
2 TB



4 TB

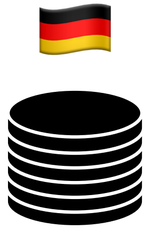


2 TB

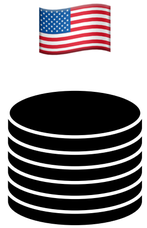


1 TB

Data

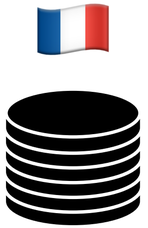


2 TB

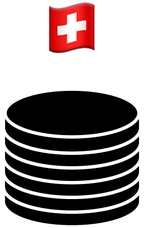


4 TB

2-Safety: Need 2 Copies

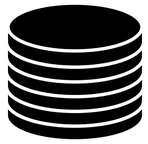


2 TB

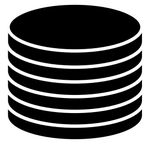


1 TB

Data

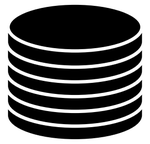


2 TB

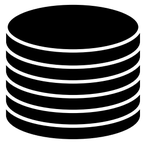


4 TB

2-Safety: Need 2 Copies



2 TB



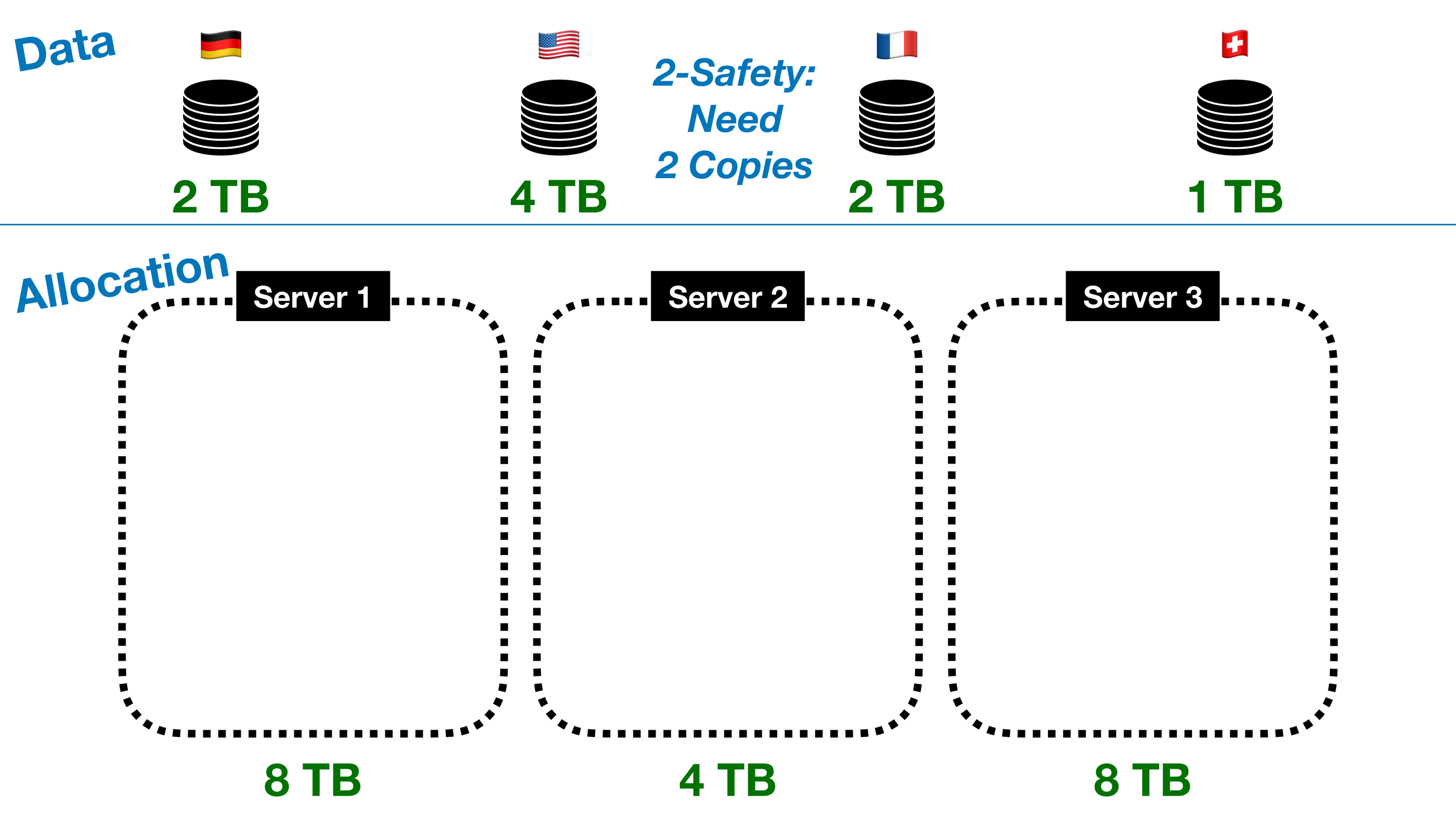
1 TB

Allocation

Server 1

Server 2

Server 3


Data
2 TB
4 TB
2-Safety:
Need
2 Copies
2 TB
1 TB
Allocation
Server 1
Server 2
Server 3
8 TB
4 TB
8 TB

Data

2 TB

4 TB

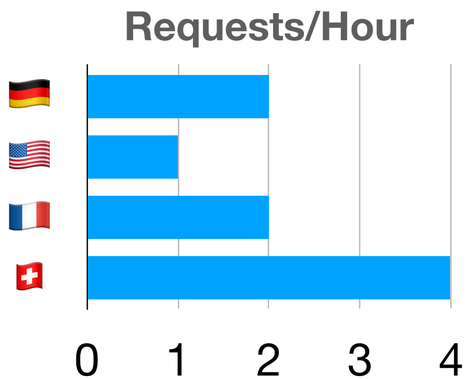
2-Safety: Need 2 Copies

2 TB

1 TB

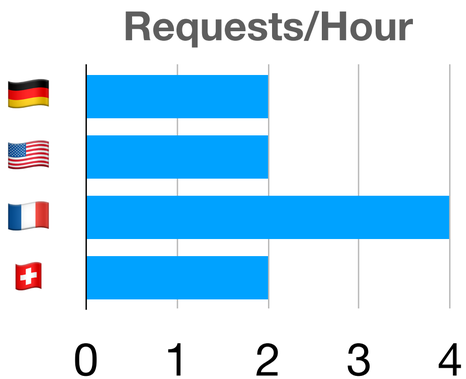
Allocation

Server 1



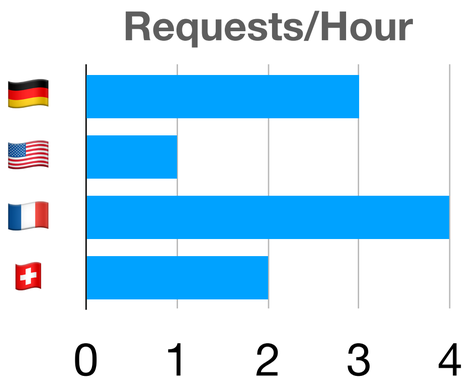
8 TB

Server 2



4 TB

Server 3



8 TB

Data

2 TB

4 TB

2-Safety: Need 2 Copies

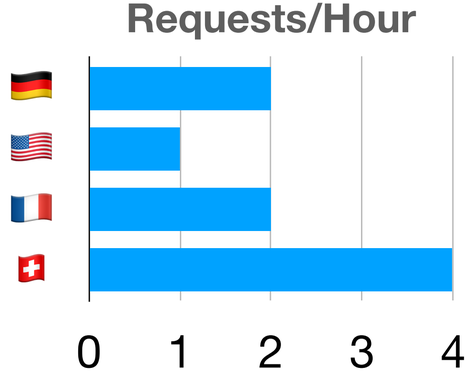
2 TB

1 TB

Allocation

Server 1

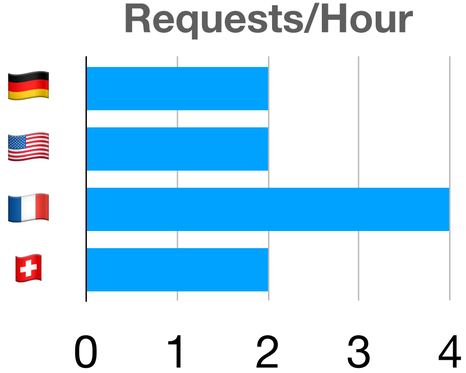
2 TB 4 TB 2 TB



8 TB

Server 2

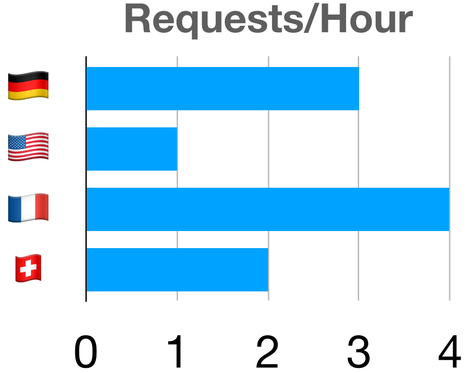
2 TB 1 TB



4 TB

Server 3

2 TB 4 TB 1 TB



8 TB

Data

2 TB

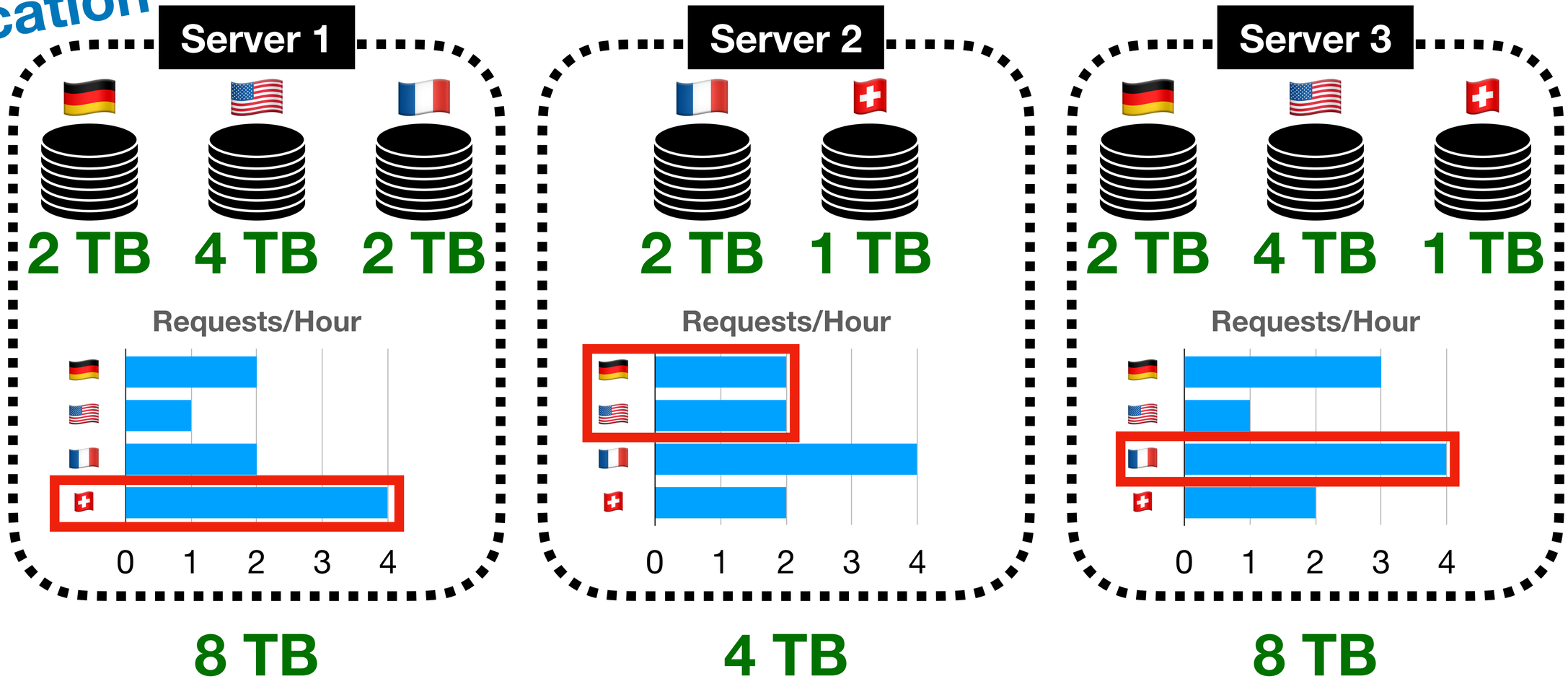
4 TB

2-Safety: Need 2 Copies

2 TB

1 TB

Allocation



Data

2 TB

4 TB

2-Safety: Need 2 Copies

2 TB

1 TB

Allocation

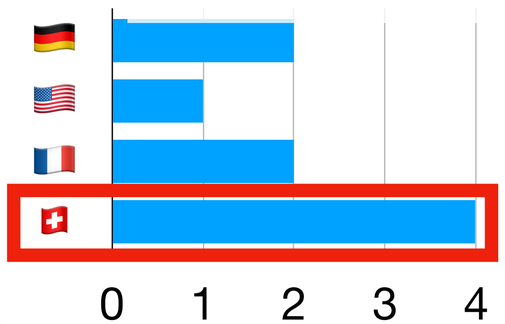
Server 1

Server 2

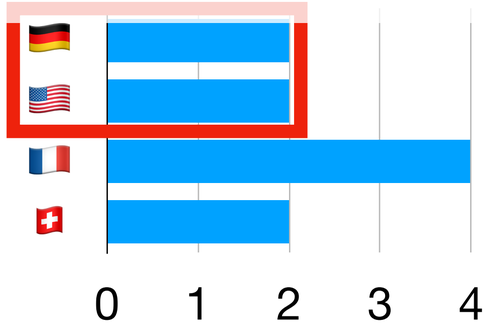
Server 3

Costs: 24

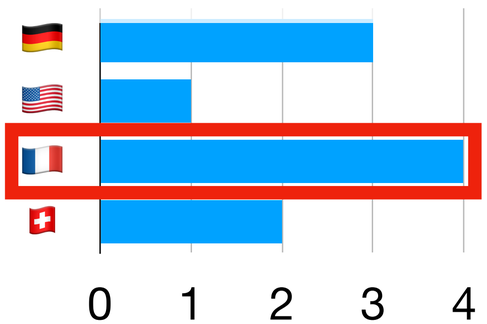
Requests/Hour



Requests/Hour



Requests/Hour



8 TB

4 TB

8 TB

Data

2 TB

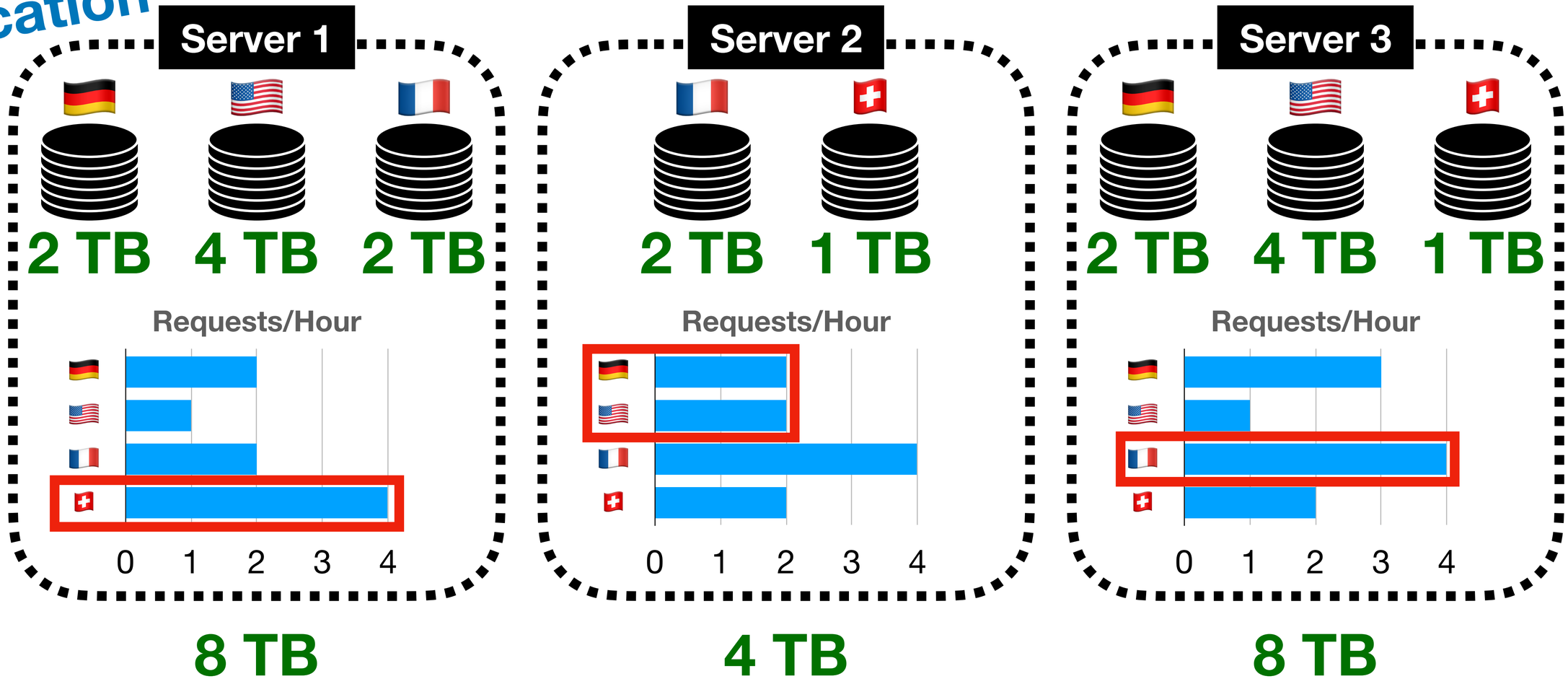
4 TB

2-Safety: Need 2 Copies

2 TB

1 TB

Allocation



Data

2 TB

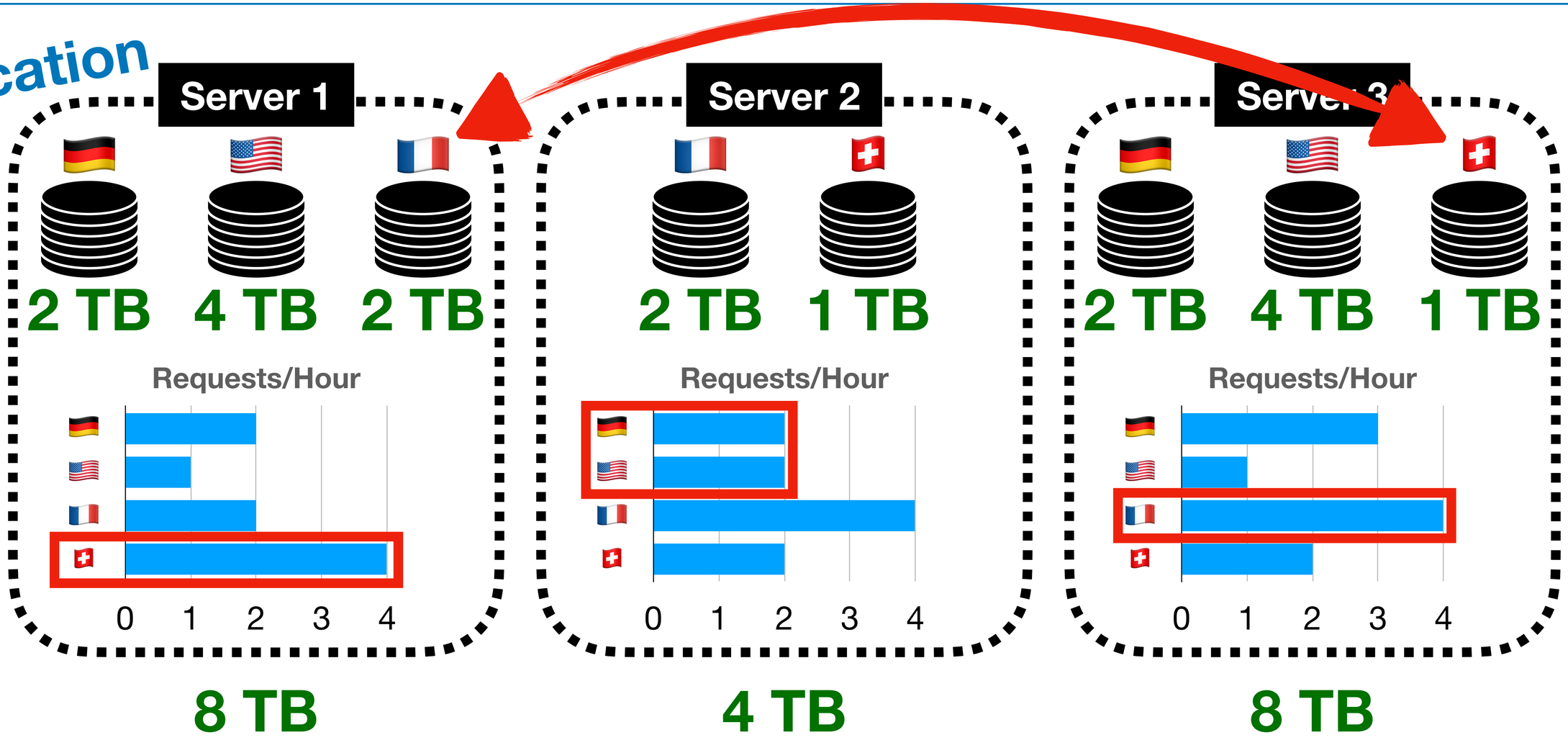
4 TB

2-Safety:
Need
2 Copies

2 TB

1 TB

Allocation



Data

2 TB

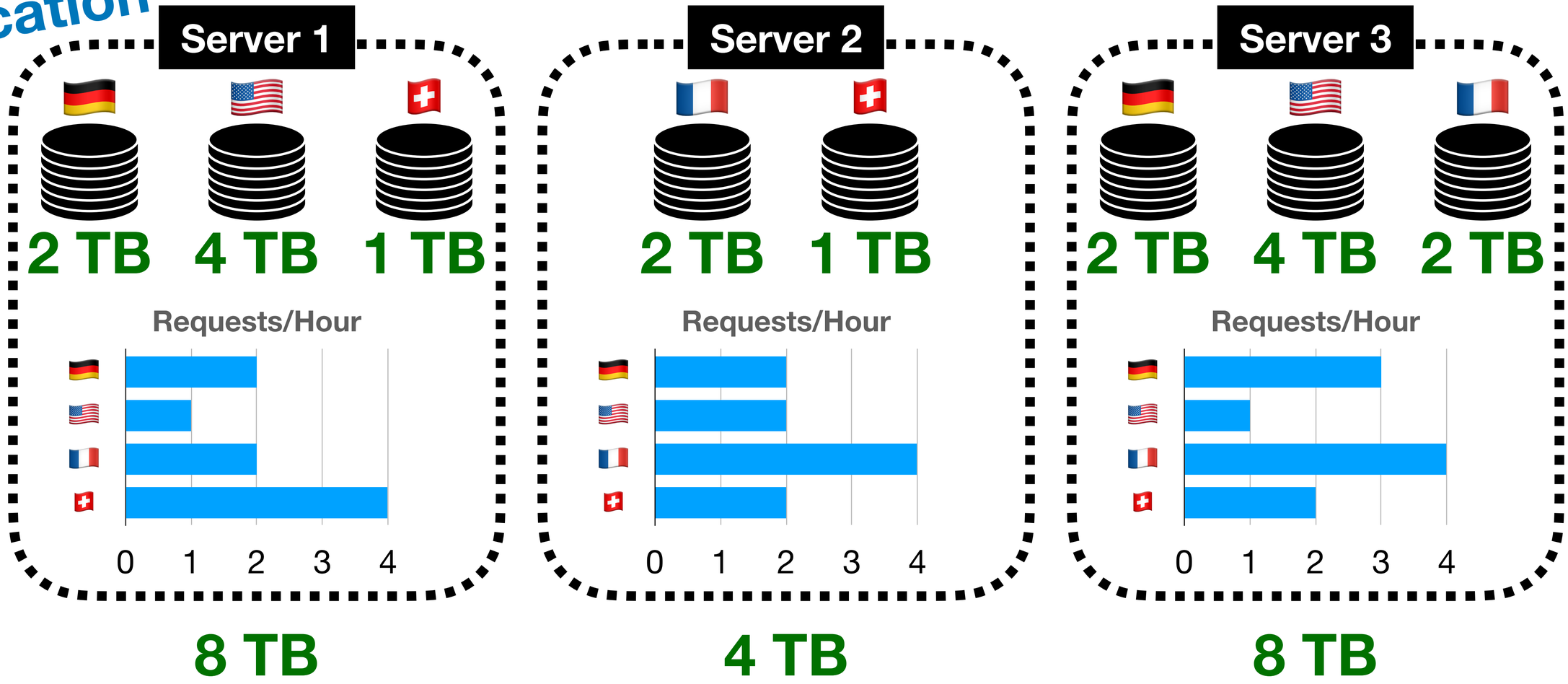
4 TB

2-Safety: Need 2 Copies

2 TB

1 TB

Allocation



Data

2 TB

4 TB

2-Safety: Need 2 Copies

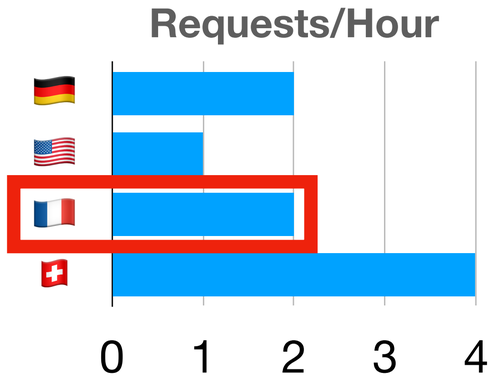
2 TB

1 TB

Allocation

Server 1

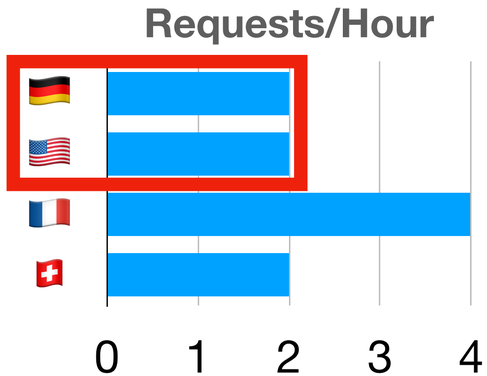
2 TB 4 TB 1 TB



8 TB

Server 2

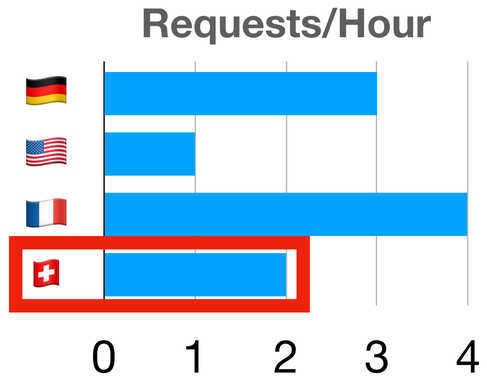
2 TB 1 TB



4 TB

Server 3

2 TB 4 TB 2 TB



8 TB

Data

2 TB

4 TB

2-Safety: Need 2 Copies

2 TB

1 TB

Allocation

Server 1

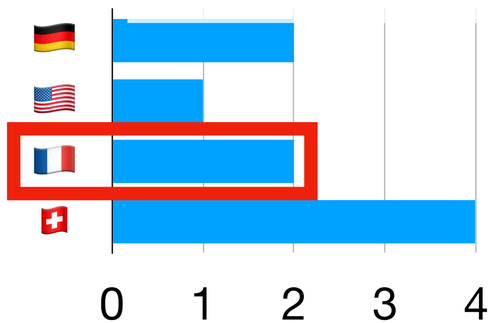
Server 2

Server 3

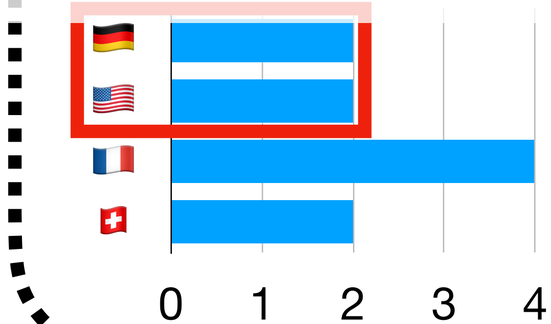
Costs: 16

2 TB

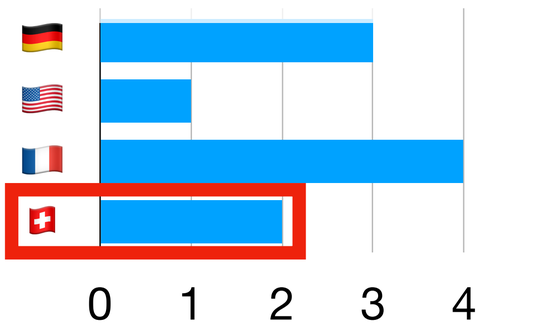
Requests/Hour



8 TB



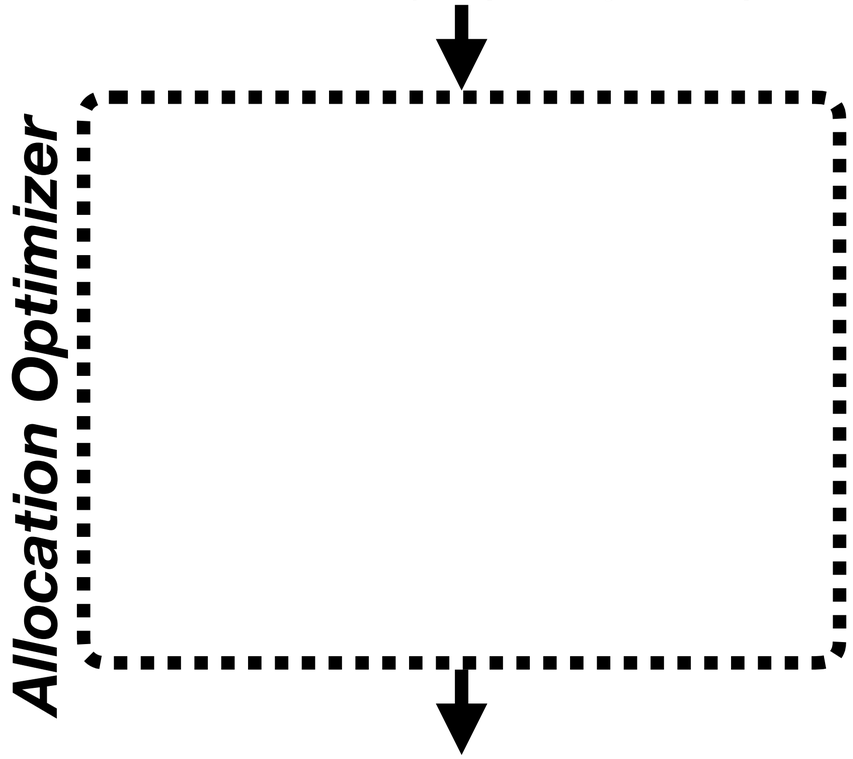
4 TB



8 TB

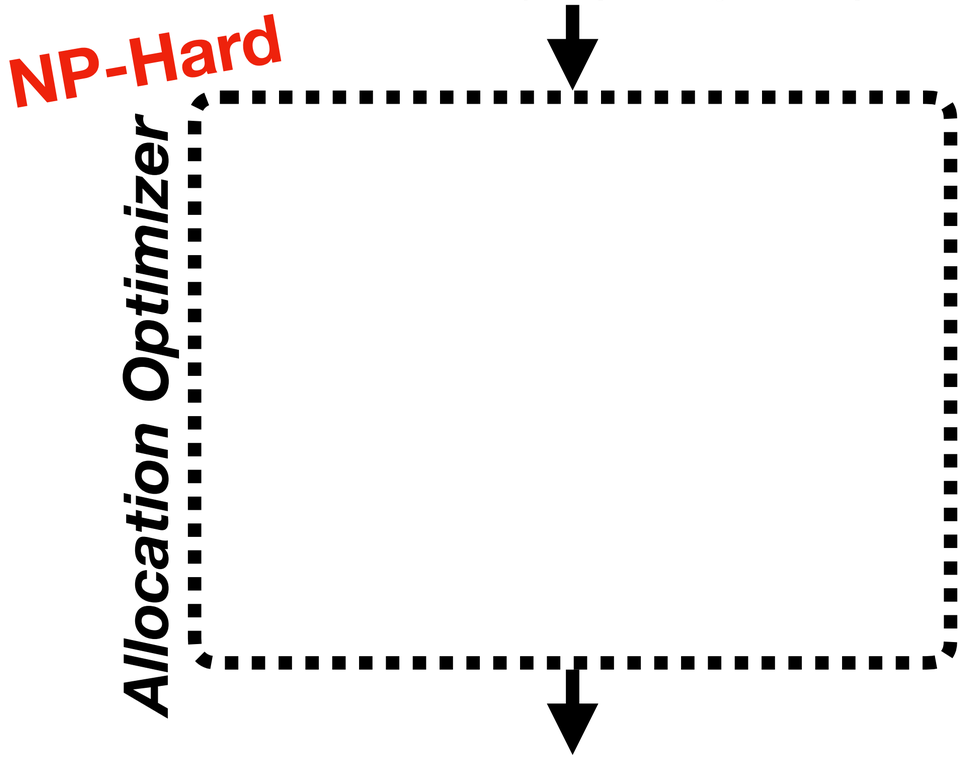
Optimal Data Allocation

- **Safety** Requirements
- **Data Properties** (Size, Remote Access Cost)
- **Server Properties** (Capacity, Request Profile)



Optimal Data Allocation

- **Safety** Requirements
- **Data Properties** (Size, Remote Access Cost)
- **Server Properties** (Capacity, Request Profile)



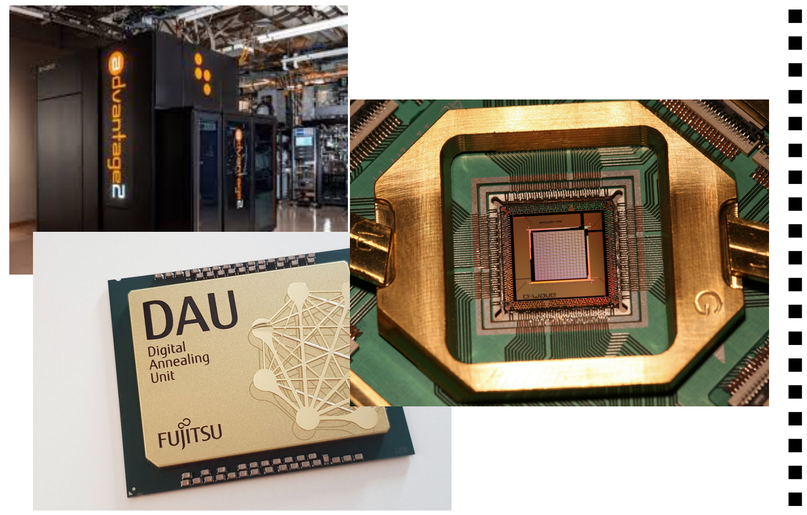
Optimal Data Allocation Approach

- **Safety** Requirements
- **Data Properties** (Size, Remote Access Cost)
- **Server Properties** (Capacity, Request Profile)

NP-Hard

Allocation Optimizer

Optimal Data Allocation



Hardware Accelerators for QUBO Problems:

Minimize $\sum_{i<j} V_i * V_j * W_{i,j}$

Optimal Data Allocation Approach

- **Safety** Requirements
- **Data Properties** (Size, Remote Access Cost)
- **Server Properties** (Capacity, Request Profile)

NP-Hard

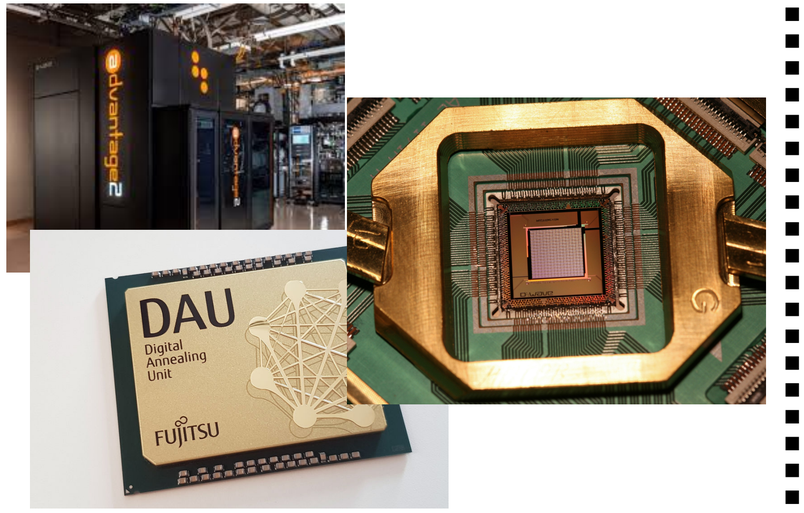
Allocation Optimizer

Transform to QUBO

Solve QUBO

Extract Data Allocation

Optimal Data Allocation

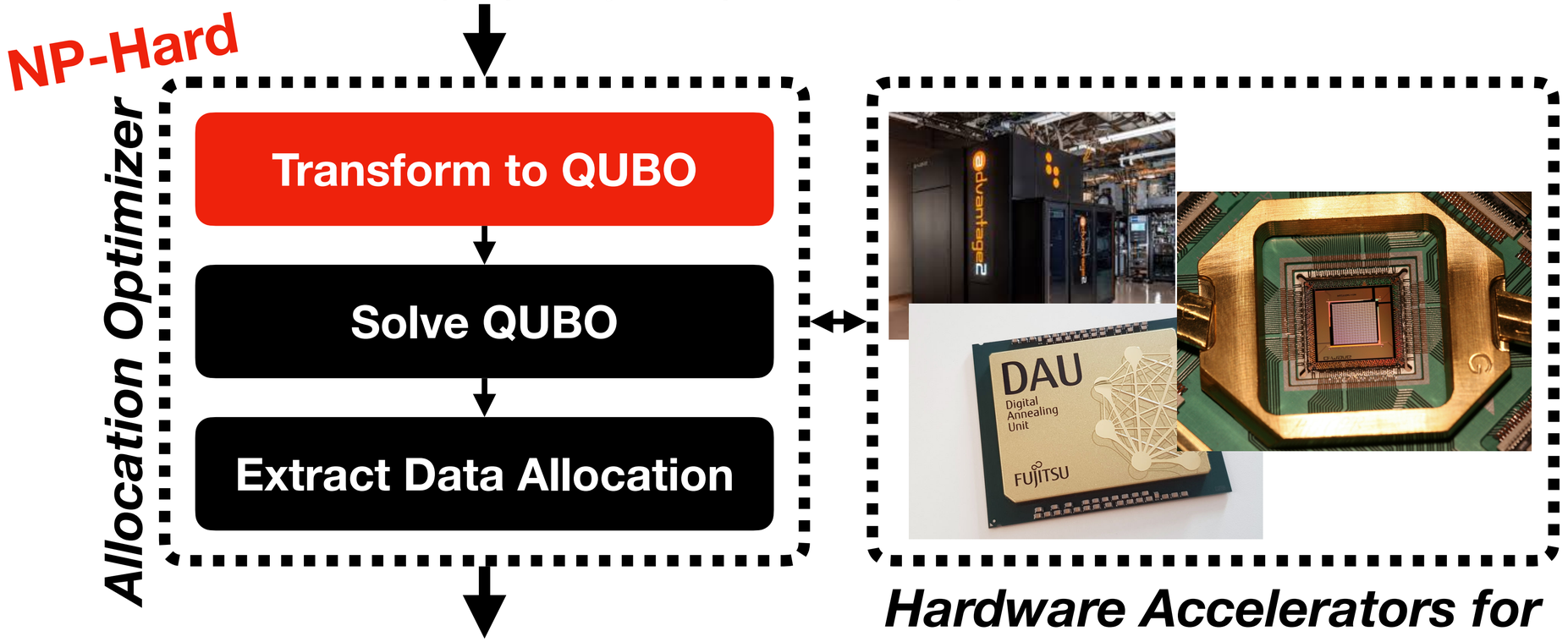


Hardware Accelerators for QUBO Problems:

Minimize $\sum_{i<j} V_i * V_j * W_{i,j}$

Optimal Data Allocation Approach

- **Safety** Requirements
- **Data Properties** (Size, Remote Access Cost)
- **Server Properties** (Capacity, Request Profile)



QUBO Formula

$\mathbf{Assigned_{p,n}}$: 1 iff partition *p* assigned to node *n*
$\mathbf{Used_{i,n}}$: 1 iff storage chunk of size 2^i used at node *n*

QUBO Formula

$\text{Assigned}_{p,n}$: 1 iff partition *p* assigned to node *n*
$\text{Used}_{i,n}$: 1 iff storage chunk of size 2^i used at node *n*

[Processing costs with current data assignments]

+ Weight * (

[>0 if less than k copies for some partitions] +

[>0 if capacity constraint violated for some nodes]

)

QUBO Formula

$\text{Assigned}_{p,n}$: 1 iff partition *p* assigned to node *n*
$\text{Used}_{i,n}$: 1 iff storage chunk of size 2^i used at node *n*

$\sum_{p \in \text{Partitions}}$ (RemoteCost_p * [# Remote retrievals for partition p])

+ Weight * (

[>0 if less than k copies for some partitions] +

[>0 if capacity constraint violated for some nodes]

)

QUBO Formula

$Assigned_{p,n}$: 1 iff partition *p* assigned to node *n*
$Used_{i,n}$: 1 iff storage chunk of size 2^i used at node *n*

$\sum_{p \in Partitions} (RemoteCost_p * (\sum_{n \in Nodes} Requests_{p,n} *$ [1 if not assigned]))

+ Weight * (

[>0 if less than k copies for some partitions] +

[>0 if capacity constraint violated for some nodes]

)

QUBO Formula

$Assigned_{p,n}$: 1 iff partition *p* assigned to node *n*
$Used_{i,n}$: 1 iff storage chunk of size 2^i used at node *n*

$$\sum_{p \in Partitions} (RemoteCost_p * (\sum_{n \in Nodes} Requests_{p,n} * (1 - Assigned_{p,n})))$$

$$+ Weight * ($$

[>0 if less than k copies for some partitions] +

[>0 if capacity constraint violated for some nodes]

$$)$$

QUBO Formula

$\text{Assigned}_{p,n}$: 1 iff partition *p* assigned to node *n*
$\text{Used}_{i,n}$: 1 iff storage chunk of size 2^i used at node *n*

$$\sum_{p \in \text{Partitions}} (\text{RemoteCost}_p * (\sum_{n \in \text{Nodes}} \text{Requests}_{p,n} * (1 - \text{Assigned}_{p,n})))$$

$$+ \text{Weight} * ($$

$$\sum_{p \in \text{Partitions}}([\#\text{Copies of partition p}] - k)^2 +$$

[>0 if capacity constraint violated for some nodes]

$$)$$

QUBO Formula

$\text{Assigned}_{p,n}$: 1 iff partition *p* assigned to node *n*
$\text{Used}_{i,n}$: 1 iff storage chunk of size 2^i used at node *n*

$$\sum_{p \in \text{Partitions}} (\text{RemoteCost}_p * (\sum_{n \in \text{Nodes}} \text{Requests}_{p,n} * (1 - \text{Assigned}_{p,n})))$$

$$+ \text{Weight} * ($$

$$\sum_{p \in \text{Partitions}}((\sum_{n \in \text{Nodes}} \text{Assigned}_{p,n}) - k)^2 +$$

[>0 if capacity constraint violated for some nodes]

)

QUBO Formula

$\text{Assigned}_{p,n}$: 1 iff partition *p* assigned to node *n*
$\text{Used}_{i,n}$: 1 iff storage chunk of size 2^i used at node *n*

$$\sum_{p \in \text{Partitions}} (\text{RemoteCost}_p * (\sum_{n \in \text{Nodes}} \text{Requests}_{p,n} * (1 - \text{Assigned}_{p,n})))$$

$$+ \text{Weight} * ($$

$$\sum_{p \in \text{Partitions}}((\sum_{n \in \text{Nodes}} \text{Assigned}_{p,n}) - k)^2 +$$

$$\sum_{n \in \text{Nodes}} ([\text{Size of data assigned to n}] - [\text{Used capacity of n}])^2$$

$$)$$

QUBO Formula

$\text{Assigned}_{p,n}$: 1 iff partition *p* assigned to node *n*
$\text{Used}_{i,n}$: 1 iff storage chunk of size 2^i used at node *n*

$$\sum_{p \in \text{Partitions}} (\text{RemoteCost}_p * (\sum_{n \in \text{Nodes}} \text{Requests}_{p,n} * (1 - \text{Assigned}_{p,n})))$$

$$+ \text{Weight} * ($$

$$\sum_{p \in \text{Partitions}}((\sum_{n \in \text{Nodes}} \text{Assigned}_{p,n}) - k)^2 +$$

$$\sum_{n \in \text{Nodes}} (\sum_{p \in \text{Partitions}} \text{Assigned}_{p,n} * \text{size}_p - \text{[Used capacity of n]})^2$$

$$)$$

QUBO Formula

$\text{Assigned}_{p,n}$: 1 iff partition *p* assigned to node *n*
$\text{Used}_{i,n}$: 1 iff storage chunk of size 2^i used at node *n*

$$\sum_{p \in \text{Partitions}} (\text{RemoteCost}_p * (\sum_{n \in \text{Nodes}} \text{Requests}_{p,n} * (1 - \text{Assigned}_{p,n})))$$

$$+ \text{Weight} * ($$

$$\sum_{p \in \text{Partitions}}((\sum_{n \in \text{Nodes}} \text{Assigned}_{p,n}) - k)^2 +$$

$$\sum_{n \in \text{Nodes}} (\sum_{p \in \text{Partitions}} \text{Assigned}_{p,n} * \text{size}_p - \sum_{i \in I[n]} 2^i * \text{Used}_{i,n})^2$$

$$)$$

QUBO Formula

$\text{Assigned}_{p,n}$: 1 iff partition *p* assigned to node *n*
$\text{Used}_{i,n}$: 1 iff storage chunk of size 2^i used at node *n*

$$\sum_{p \in \text{Partitions}} (\text{RemoteCost}_p * (\sum_{n \in \text{Nodes}} \text{Requests}_{p,n} * (1 - \text{Assigned}_{p,n})))$$

$$+ \text{Weight} * ($$

$$\sum_{p \in \text{Partitions}}((\sum_{n \in \text{Nodes}} \text{Assigned}_{p,n}) - k)^2 +$$

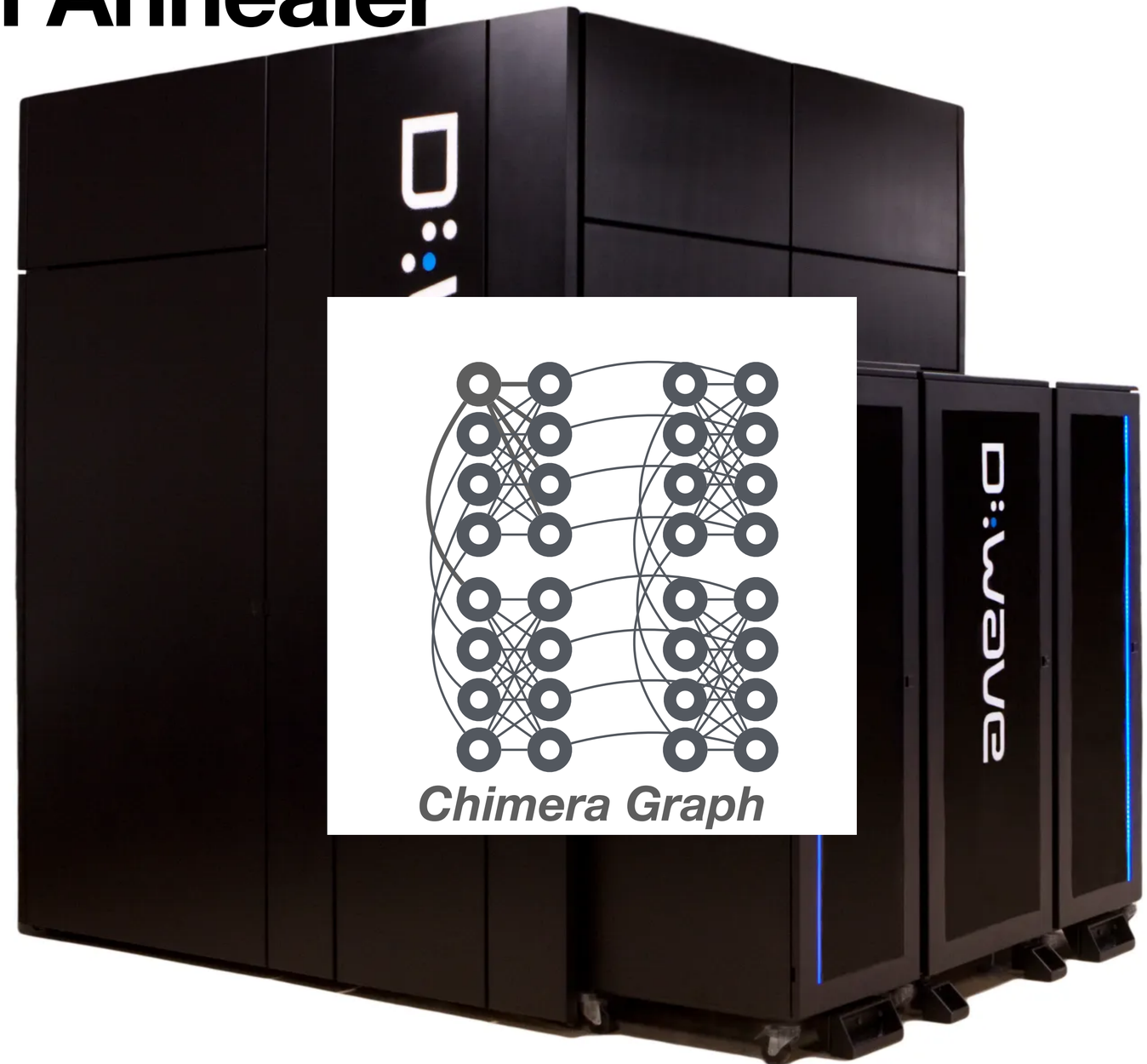
$$\sum_{n \in \text{Nodes}} (\sum_{p \in \text{Partitions}} \text{Assigned}_{p,n} * \text{size}_p - \sum_{i \in I[n]} 2^i * \text{Used}_{i,n})^2$$

$$)$$

Asymptotic growth of required qubits?

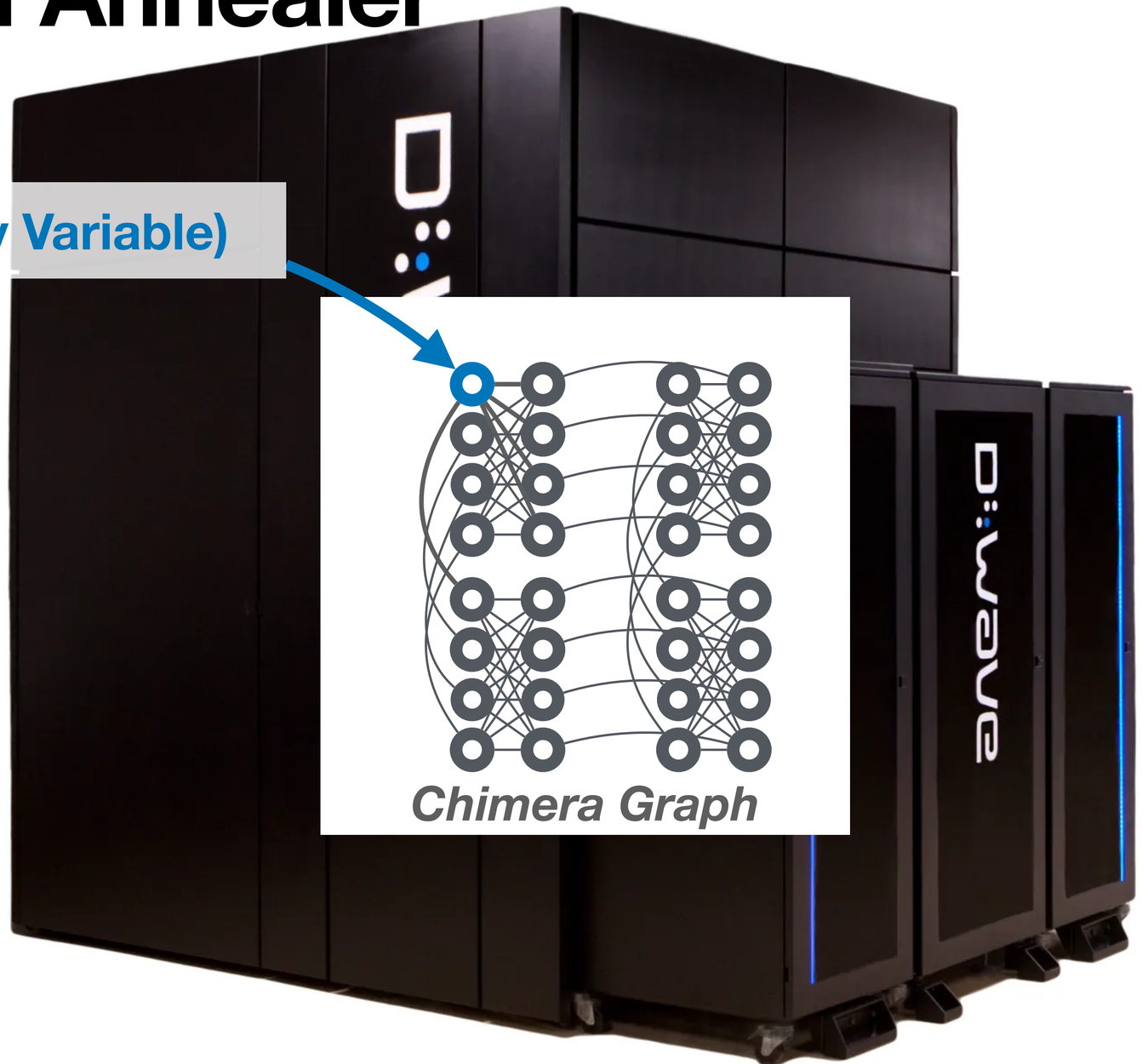
Quantum Annealer

Quantum Annealer



Quantum Annealer

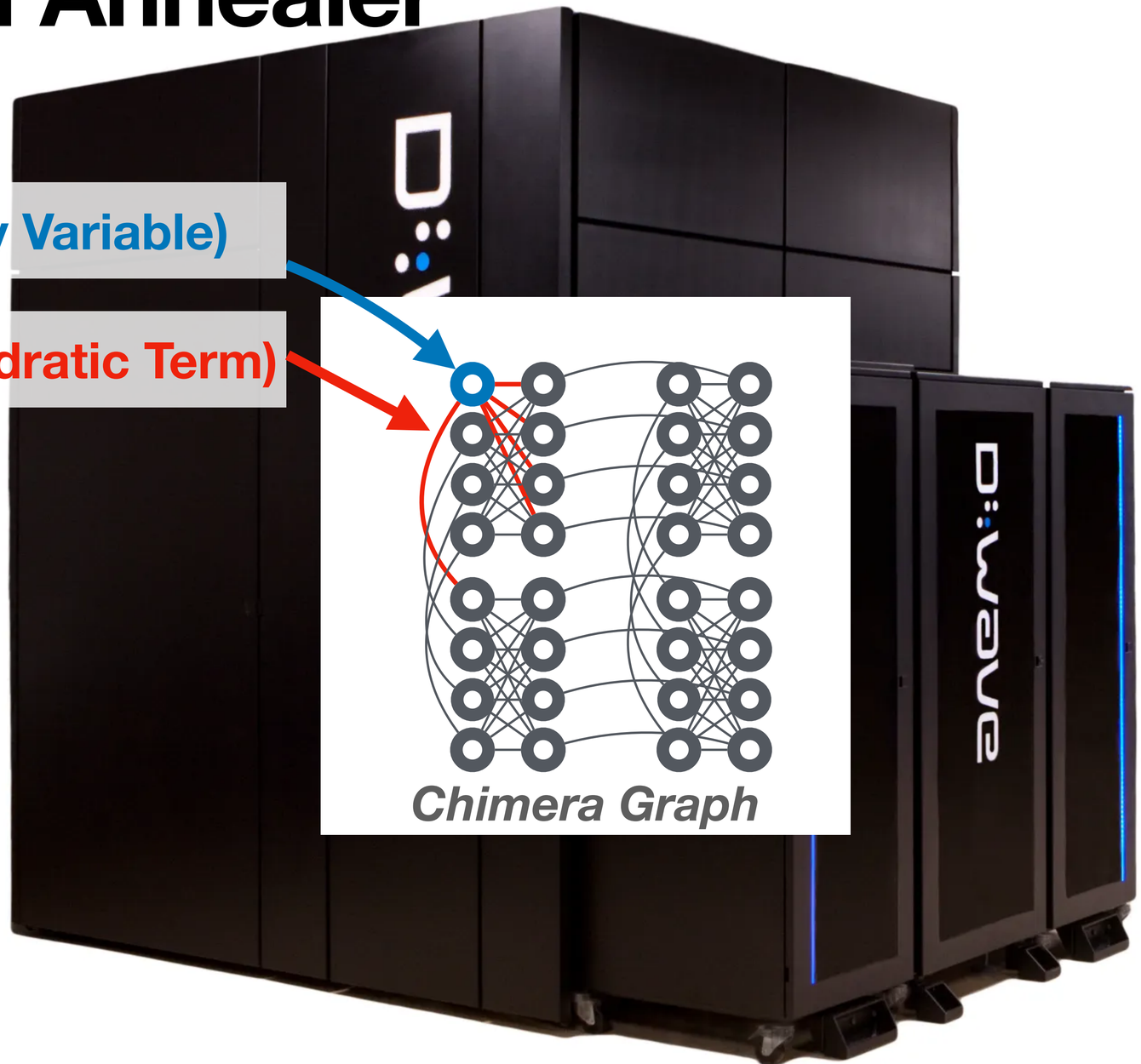
Qubit (Binary Variable)



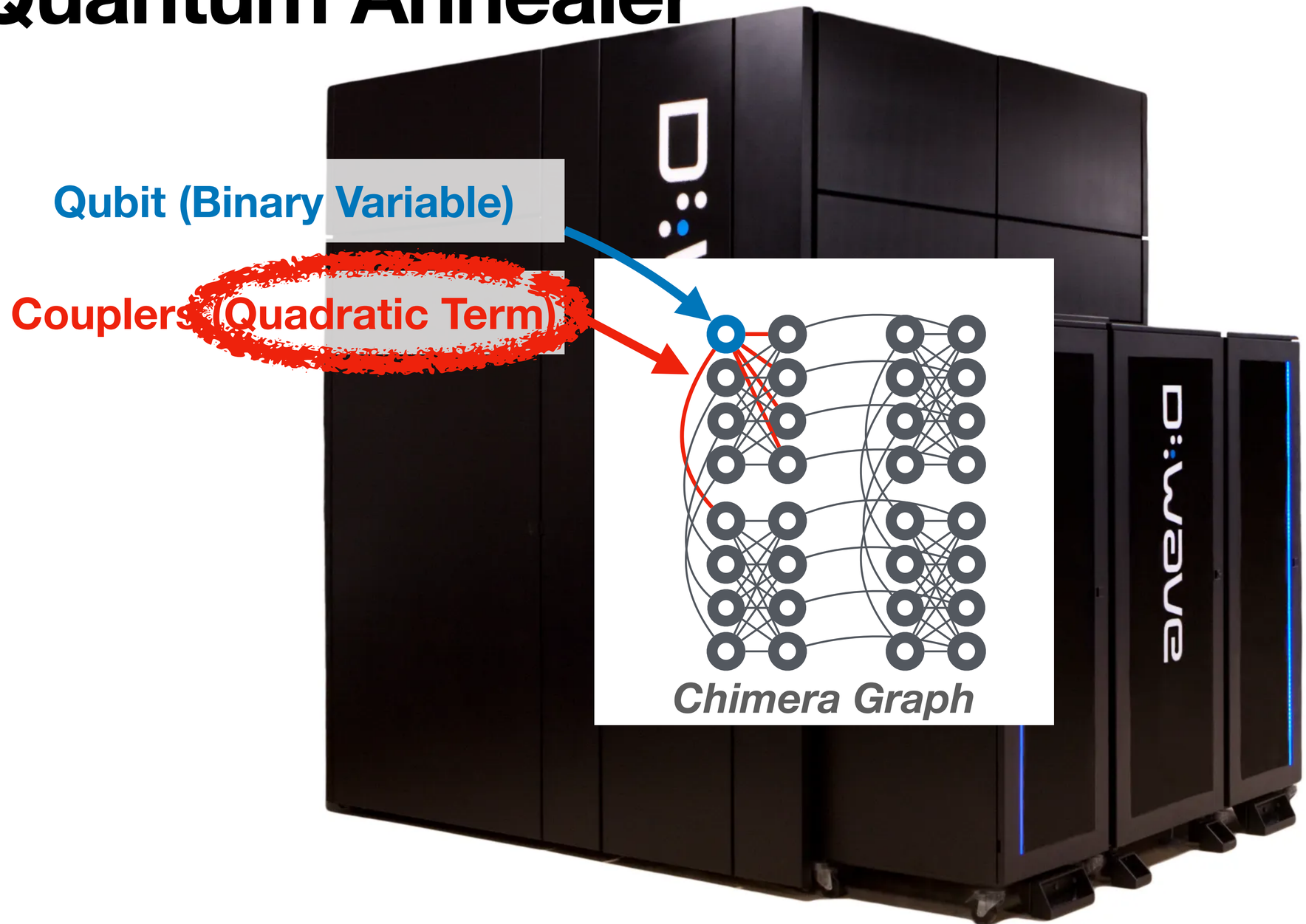
Quantum Annealer

Qubit (Binary Variable)

Couplers (Quadratic Term)



Quantum Annealer



QUBO Formula

$\text{Assigned}_{p,n}$: 1 iff partition *p* assigned to node *n*
$\text{Used}_{i,n}$: 1 iff storage chunk of size 2^i used at node *n*

$$\sum_{p \in \text{Partitions}} (\text{RemoteCost}_p * (\sum_{n \in \text{Nodes}} \text{Requests}_{p,n} * (1 - \text{Assigned}_{p,n})))$$

$$+ \text{Weight} * ($$

$$\sum_{p \in \text{Partitions}}((\sum_{n \in \text{Nodes}} \text{Assigned}_{p,n}) - k)^2 +$$

$$\sum_{n \in \text{Nodes}} (\sum_{p \in \text{Partitions}} \text{Assigned}_{p,n} * \text{size}_p - \sum_{i \in I[n]} 2^i * \text{Used}_{i,n})^2$$

$$)$$

#Qubits ∈ Ω(#Quadratic Terms)

QUBO Formula

$\text{Assigned}_{p,n}$: 1 iff partition *p* assigned to node *n*
$\text{Used}_{i,n}$: 1 iff storage chunk of size 2^i used at node *n*

$$\sum_{p \in \text{Partitions}} (\text{RemoteCost}_p * (\sum_{n \in \text{Nodes}} \text{Requests}_{p,n} * (1 - \text{Assigned}_{p,n})))$$

$$+ \text{Weight} * ($$

Quadratic terms between all variables associated with the same partition

$$\sum_{p \in \text{Partitions}} (\sum_{n \in \text{Nodes}} \text{Assigned}_{p,n}) - k)^2 +$$

$$\sum_{n \in \text{Nodes}} (\sum_{p \in \text{Partitions}} \text{Assigned}_{p,n} * \text{size}_p - \sum_{i \in I[n]} 2^i * \text{Used}_{i,n})^2$$

$$)$$

#Qubits ∈ Ω(#Quadratic Terms)

QUBO Formula

$Assigned_{p,n}$: 1 iff partition *p* assigned to node *n*
$Used_{i,n}$: 1 iff storage chunk of size 2^i used at node *n*

$$\sum_{p \in Partitions} (RemoteCost_p * (\sum_{n \in Nodes} Requests_{p,n} * (1 - Assigned_{p,n})))$$

$$+ Weight * ($$

Quadratic terms between all variables associated with the same partition

$$\sum_{p \in Partitions} (\sum_{n \in Nodes} Assigned_{p,n}) - k)^2 +$$

$$\sum_{n \in Nodes} (\sum_{p \in Partitions} Assigned_{p,n} * size_p - \sum_{i \in I[n]} 2^i * Used_{i,n})^2$$

$$)$$

$$\#Qubits \in \Omega(\#Quadratic\ Terms)$$

$$= \#Nodes^2$$

QUBO Formula

$Assigned_{p,n}$: 1 iff partition *p* assigned to node *n*
$Used_{i,n}$: 1 iff storage chunk of size 2^i used at node *n*

$$\sum_{p \in Partitions} (RemoteCost_p * (\sum_{n \in Nodes} Requests_{p,n} * (1 - Assigned_{p,n})))$$

$$+ Weight * ($$

Quadratic terms between all variables associated with the same partition

$$\sum_{p \in Partitions} (\sum_{n \in Nodes} Assigned_{p,n}) - k)^2 +$$

$$\sum_{n \in Nodes} (\sum_{p \in Partitions} Assigned_{p,n} * size_p - \sum_{i \in I[n]} 2^i * Used_{i,n})^2$$

$$)$$

Quadratic terms between all variables associated with the same node

$$\#Qubits \in \Omega(\#Quadratic\ Terms)$$

$$= \#Nodes^2$$

QUBO Formula

$\text{Assigned}_{p,n}$: 1 iff partition *p* assigned to node *n*
$\text{Used}_{i,n}$: 1 iff storage chunk of size 2^i used at node *n*

$$\sum_{p \in \text{Partitions}} (\text{RemoteCost}_p * (\sum_{n \in \text{Nodes}} \text{Requests}_{p,n} * (1 - \text{Assigned}_{p,n})))$$

$$+ \text{Weight} * ($$

Quadratic terms between all variables associated with the same partition

$$\sum_{p \in \text{Partitions}} (\sum_{n \in \text{Nodes}} \text{Assigned}_{p,n}) - k)^2 +$$

$$\sum_{n \in \text{Nodes}} (\sum_{p \in \text{Partitions}} \text{Assigned}_{p,n} * \text{size}_p - \sum_{i \in I[n]} 2^i * \text{Used}_{i,n})^2$$

$$)$$

Quadratic terms between all variables associated with the same node

$$\#\text{Qubits} \in \Omega(\ \#\text{Quadratic Terms})$$
$$= \#\text{Nodes}^2 + (\#\text{Partitions} + \log(\text{capacity}))^2$$

Next Steps

- Domain-specific methods for **physical embedding**
- Reduce **asymptotic growth** of required qubits
- Obtain **empirical results**

Conclusion

- Focus on optimal **data allocation**
- Presented **transformation** to QUBO
- Analyzed **growth** of required qubits